Rational Choice Under an Imperfect Ability To Choose

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We consider an individual who lacks the information-processing capacity required for a direct comparison of all feasible allocations. Instead of finding at once a best allocation, the individual myopically adjusts his current allocation toward higher utility. The individual makes adjustment errors inversely proportional to his ability to choose. We compare the stationary state of this process with the standard model. We see how an imperfect ability to choose modifies both positive and normative predictions of the standard model and how the standard model can be obtained from our more general one as the special case corresponding to perfect ability. (*JEL* D11)

The standard model of consumer behavior asserts that individuals have unlimited information-processing capacity, which allows them to solve their choice problem in a strictly optimal manner irrespective of the difficulty of the problem. However, existing evidence indicates that individuals have only limited information-processing capacity.[1] In consequence, when deciding among alternative courses of action, individuals use simple, local and myopic choice procedures which adapt choice behavior to their capacity limitations (see e.g., Allen Newell and Herbert A. Simon, 1972). The quality of choice procedures used reflects the *ability to choose*. Since such procedures are imperfect, they cause random errors when the choice problem is of sufficient complexity. Thus, within our framework, individual choice behavior can only be determined up to a probability distribution which serves as a "black box" to summarize complex behavioral aspects of the individual. Such ambiguity in choice predictions is deeply rooted in the real world. It arises because individuals may not know their preferences perfectly well, and it differs from that generated by an uncertain future in the context of rational choice (von Neumann and Morgenstern, 1944). Of course, variance may well arise through uncertainty about the future. But it may also arise as an inherent property of choice behavior even when the future can be known (James G. March, 1978 pp. 598–99).

We consider the choice problem of an individual who does not have the information-processing capacity required for a direct comparison of all feasible allocations. Instead of finding at once a best allocation

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[1]This observation has given rise to information-processing theories of choice, which are dominant in the behavioral sciences (see e.g., James R. Bettman, 1979; Daniel Kahneman et al., 1982). In those theories, individuals acquire information from various sources in their environment, which they perceive, interpret, and evaluate drawing upon past experience and upon the context in which they obtained it. Thus information-processing theories are not just about what individuals know, but also about how individuals use what they know. In economics, such processing-capacity limitations have been expressed as a "competence–difficulty" gap (Ronald A. Heiner, 1983), whereby the competence of individuals to solve a choice problem does not match the difficulty of that problem.

as in the standard model of consumer behavior, the individual adjusts myopically his current consumption in order to improve utility. Beginning with some initial commodity stock, the individual spends at a constant rate which defines periods corresponding to one unit of expenditure. The individual's commodity stock changes between periods following the rates at which he consumes the various commodities on the one hand, and his purchasing decisions on the other. At the beginning of each period, the individual spends the unit of money currently available on a single commodity. The problem now is to find a commodity which gives the highest marginal utility. Thus the individual reduces the unmanageable problem of comparing all feasible consumption bundles to a manageable problem of comparing the utility increments derived by adding in turn an amount of unit value to each current commodity stock. In principle, all that is necessary to solve this problem is knowledge about marginal abilities at a point, rather than about the utility function itself.

Following the information-processing approach to choice, we allow for the possibility that the individual makes errors in comparing marginal utilities. This creates a difference between perceived and true marginal utilities, which decreases as the ability to choose increases. For a particular degree of ability, the above choice procedure corresponds to a particular stationary state in consumption. As the ability to choose increases, the stationary level of utility also increases. At the limit, where the ability to choose becomes perfect, the stationary state coincides with the solution of the standard consumer choice problem. Thus, for different degrees of ability to choose, our model predicts a dispersion of choice decisions even in cases where rational individuals have identical preferences, endowments, and access to information—a phenomenon which has been observed experimentally (R. J. Herrnstein, 1991). In the context of the standard consumer choice model, such dispersion must signify irrational behavior.

When the individual can choose to improve his ability, he faces a trade-off between the level of ability and the income left for consumption. Under these conditions, a maximum feasible level of utility can be reached, provided the individual does not make errors in improving his ability. In the presence of such errors, our framework leads to a failure of both fundamental welfare theorems which does not arise from externalities or a lack of information, but rather because the individual does not have the information-processing capacity necessary to equate the marginal cost of improving ability with the corresponding marginal benefit. In such cases, paternalism may be justified.

We next examine the possibility that random errors are biased. A natural interpretation for such errors can be found in the context of image advertisement. In most economic models, such advertisement has been treated as a good that yields either information or utility. We, on the other hand, focus on the manipulative characteristics of image advertisement which can be either good (if error bias encourages consumption that increases utility) or bad (if it encourages consumption that decreases utility). The impact of such advertisement becomes smaller for higher ability to choose until, when the ability to choose is perfect, manipulative advertisement is rendered completely ineffective. It follows that, when individuals are perfect instruments of choice and have fixed preferences, as in standard economic theory, there is no conceptual basis for manipulative advertisement: individuals either benefit from it or eventually discard it. In contrast, an imperfect ability to choose does not negate existing economic approaches to advertisement. For example, image advertisement can enter the utility function as a complement to the good advertised (Gary Becker and Kevin M. Murphy, 1993) and, at the same time, bias the perception of true utility when the individual has an imperfect ability to choose. Our approach provides a rationale for laws against false advertisement, for the regulation of advertisement of commodities for which errors may become dangerous, and for advertisement aimed specifically at low-ability groups such as young children.

Throughout the paper, we provide examples about how an imperfect ability to choose modifies both the positive and normative implications of the standard consumer model.[2] In the final section we provide a model of socially optimal product differentiation with fixed prices in which normative implications vary systematically with the degree of ability to choose. When the ability to choose is perfect, we derive the standard solution. The optimal products become increasingly similar for lower ability until, below a certain level, there remains only one optimal product. Besides providing a formalization of the familiar idea that there can be too much product variety, this example provides another case in which the implications of variable ability to choose cannot be dismissed as insignificant.

I. Myopic Adjustments

If an individual can evaluate and rank all feasible consumption bundles, the description of the choice procedure adopted becomes trivial: the individual simply ranks all feasible consumption bundles according to his preferences and chooses a most preferred one. Under these circumstances, the choice problem can be expressed in the familiar, compact form:

$$(1) \qquad \max_{Q_1,\ldots,Q_n} U(Q_1,\ldots,Q_n)$$

subject to

$$\sum_{i=1}^{n} p_i Q_i = Y$$

where $U(\cdot)$ is the ordinal utility function, p_i and Q_i are the price and the consumption of the ith commodity, and Y is income. If, however, the individual does not have the ability for making the global comparisons required to solve a problem (1), then he is bound to adjust his choice behavior accordingly. For example, if we confronted you with a complete display of all the consumption choices you made last month, together with a reasonable (for you) alternative display of consumption choices, we doubt that you would be prepared to undertake the global comparisons required for expressing preference between those two consumption bundles. You would be prepared, however, to discuss the advantages of choosing this brand of cereal over that brand, or the excessive amounts you spent last month on entertainment. More generally, excepting some well-defined cases in which the problem of choosing among a small number of alternatives is independent of everything else, consumption choices rarely, if at all, concern entire consumption bundles. Typically *consumption choices are about elements of the consumption bundle and about consumption-bundle change*. In this sense, consumption choices are typically *local* rather than global.[3]

One main goal of our paper is to model a sequence of consumption choices that describes local consumption-bundle adjustments. We imagine that this sequence occurs during a time interval of length T. At the beginning of the time interval, the individual receives a fixed income Y, which it spends on the n commodities. To fix ideas, let T represent a month and let Y represent the salary received at the beginning of the month. There is no borrowing or saving across months. We assume that spending occurs at a uniform rate $y = Y/T$ which determines the partitioning of the month into periods of length Δt corresponding to one unit of expenditure:

$$(2) \qquad y\Delta t = 1.$$

[2]See also George A. Akerlof and Janet L. Yellen (1985) for a discussion about how even small errors in consumption decisions can have large effects on equilibria.

[3]This is closer to a Marshallian point of view (Alfred Marshall, 1920 [Vol. 3, Chapter III]). It is interesting to note that John R. Hicks, a protagonist in the establishment of ordinal utility theory, later recognised the descriptive accuracy of the old consumer theory (Hicks, 1976 pp. 137–38).

At the beginning of each period $[t, t+\Delta t]$ $(t = 0, \Delta t, 2\Delta t, \ldots, (Y-1)\Delta t)$ the individual spends the available unit on a single commodity. We thus replace the global choice of one among an infinite number of commodity combinations in the feasible consumption set with a sequence of simpler, local decisions about spending one unit of money on one out of n discrete alternatives.

Let $Q_{i,t}$ represent the stock of commodity i at the beginning of the period $[t, t+\Delta t]$. During that period, this stock is consumed at a uniform rate $q_{i,t}$ determined at the beginning of the period. The continuous decrease of the stock during that period is given by

$$(3) \qquad Q_{i,t}(\tau) = Q_{i,t}(1 - c_i\tau)$$

where $0 \leq \tau \leq \Delta t$, $c_i\tau \leq 1$, and c_i is a constant that determines how fast one unit of commodity i is depleted. It follows that the uniform consumption rate for commodity i during the period $[t, t+\Delta t]$ is given by

$$(4) \qquad q_{i,t} = c_i Q_{i,t}.$$

If, at the beginning of the next period, the individual spends the newly accumulated unit of money on commodity i, the corresponding stock increases by $1/p_i$, and the consumption rate increases by c_i/p_i. Thus, although consumption rates remain fixed within periods, they can vary between periods according to the size of stock owned at the beginning of each period, where a larger stock implies a higher rate of consumption.

The problem of the individual at the beginning of each period is to spend the unit of money on a commodity i that generates the largest utility increment. Utility in our model is determined by the rate of consumption, rather than by the stock itself: if you do not use what you have, you do not derive utility from it. Let $u(q_{1,t}, \ldots, q_{n,t})$ represent the individual's perception of the utility derived from the n consumption rates. We can now replace the global choice problem (1) with the following sequence of local choice problems:

at the beginning t of each period in $[0, T]$ spend $y\Delta t$ on a commodity i such that

$$(5) \qquad \Delta_{i,t}u > \max\{\Delta_{j,t}u;\ j \neq i\}$$

where $\Delta_{i,t}u$ is the utility increment derived from buying commodity i at t. If we assume that $u(\cdot)$ represents the utility derived from the n consumption rates during the period $[t, t+\Delta t]$, then (5) is akin to *melioration*, "(the) process of comparing the rates of return and shifting toward the alternative that is currently yielding the better return..." (Herrnstein, 1991 p. 361). This is a widely observed type of choice behavior in various species, including humans. For example, in series of experiments designed to determine whether individuals maximize utility in the global sense or meliorate, Herrnstein found strong evidence in support of the second alternative.[4] Individual behavior, at least in the environment provided by those experiments, was consistently myopic as implied by (5). Herrnstein uses his findings as evidence that individuals do not maximize utility. In contrast, we shall argue that the Herrnstein experiments are consistent with utility maximization subject to error.

II. Errors of Perception

We have proposed the sequence of local choice problems as a more accurate description of individual consumption behavior because we believe that individuals do not have the ability to make the global comparisons required for solving problem (1). In

[4]There were three types of experiment, involving a sequence of binary choices where current return depended on previous choices in a systematic way which could be understood given the information available. Maximizing utility would require taking into account the consequences of the current choice on future returns. In the first type of experiment, utility maximization predicted indifference between alternatives, while melioration predicted a unique outcome. In the second type, the predictions were reversed. Finally, in the third type, both choice procedures predicted diammetrically opposed, unique outcomes. In every type of experiment, individuals tended to meliorate quite strongly.

the Herrnstein experiments, such behavior led to suboptimal choice outcomes because the alternative yielding the better strictly current return is not necessarily the alternative yielding the better current return after taking into account the consequences of current choice on future returns. This is just one out of many ways people make errors along a choice procedure such as (5).[5] In general, those errors can be defined as

$$\Delta_{i,t}u - \Delta_{i,t}v = \varepsilon_{i,t} \tag{6}$$

where $\Delta_{i,t}u$ denotes *perceived* utility increments and $\Delta_{i,t}v$ denotes *true* utility increments. This distinction is fundamental for our paper. Since the ways people can make errors are many and unpredictable, we assume that $\varepsilon_{i,t}$ is random. Its variance reflects the ability of the individual to choose, where smaller variance implies higher ability. At the limit, where perceived and true utility increments coincide, we say that the ability to choose is perfect.

Our objective is to study the outcomes of (5) for consumer choice by recognizing explicitly that individuals make errors. Substituting (6) in (5), we have the following:

> *at the beginning t of each period in* $[0,T]$ *spend* $y\Delta t$ *on a commodity i such that*

$$\Delta_{i,t}v + \varepsilon_{i,t} > \max\{\Delta_{j,t}v + \varepsilon_{j,t};\ j \neq i\}. \tag{7}$$

In the context of problem (1), this choice procedure portrays an individual who applies a gradient process to climb myopically toward the peak of the intersection between his utility surface and the hyperplane that describes the fixed amount of resources available. If the individual does not make errors, since his true utility surface has a single peak, the outcome of this procedure will also maximize utility on average.

Since errors are random, individual choices in (7) can be determined only up to a probability distribution. Accordingly,

$$\mathbb{P}_{i,t} \equiv \Pr\{\Delta_{i,t}v + \varepsilon_{i,t} > \max\{\Delta_{j,t}v + \varepsilon_{j,t};\ j \neq i\}\} \tag{8}$$

denotes the probability that the individual will spend the unit of money on commodity i at t. We shall first examine the simple case in which error variance remains the same over time. If those errors are independently and identically distributed[6] and if the preference ordering of the individual is invariant under uniform expansions of the choice set then, according to theorem 6 in John I. Yellott (1977), the random errors are double-exponential.[7] Under these circumstances, we can write the random errors as

$$\varepsilon_{i,t} = \frac{1}{\mu}\varepsilon \qquad \mu > 0 \tag{9}$$

where $1/\mu$ is the dispersion parameter of $\varepsilon_{i,t}$, and where ε is double-exponential with a zero location parameter and unit

[5] Following information-processing theories, we can imagine that the individual uses information from the environment as an input to an individual-specific processing technology and produces decisions. The processing technology will reflect the individual's investment for improving his processing capacity through, for example, education. However, it will also reflect inherent characteristics of the individual, such as perceptiveness, intelligence, quality of memory, and so forth, which make some individuals better instruments of choice than others. In this framework, errors can arise from incomplete information, the distortion of information during processing, or wrong deductions. The last possibility has been called a "lack of logical omniscience" by Barton L. Lipman (1992). Better processing technology implies smaller errors on average.

[6] A justification for stochastic independence can be made if a commodity is understood to represent a distinct class of differentiated products sold at the same price by a large number of competing firms. There is no restriction on the entry or exit of firms, or on the introduction of new products and discontinuation of old ones. We imagine that the individual draws a random combination of products of unit value belonging to the same commodity. Since the choice environment is in continuous flux, the individual can make errors in predicting the consumption effect of the particular draw on utility; and since every draw consists of a random combination, those errors can be independent.

[7] A uniform expansion of a set can be obtained by replicating every element in the set the same number of times.

dispersion parameter. Then, following Daniel McFadden (1974), the marginal allocation probabilities (8) are given by the multinomial logit model:

$$(10)\qquad \mathbb{P}_{i,t}=\frac{\exp(\mu\Delta_{i,t}v)}{\sum_{j=1}^{n}\exp(\mu\Delta_{j,t}v)}.$$

Since errors become smaller for larger μ, this parameter can be interpreted as the ability of the individual to choose, where a larger μ signifies higher ability. We also have

$$(11)\qquad \lim_{\mu\to 0}\mathbb{P}_{i,t}=\frac{1}{n}$$

$$\lim_{\mu\to\infty}\mathbb{P}_{i,t}=\begin{cases}1 & \text{if } \Delta_{i,t}v>\max\{\Delta_{j,t}v,\ j\neq i\}\\ 0 & \text{otherwise.}\end{cases}$$

When there is no ability to choose, discrete choices are equiprobable, irrespective of differences in the true marginal value of alternatives; and when the ability to choose is perfect, the best choice is made with certainty. More generally, the individual's ability to choose is reflected by the distribution of marginal allocation probabilities around alternatives of higher marginal utility. As the ability to choose increases, the distribution of marginal allocation probabilities tightens around better alternatives until, at the limit, the individual adjusts only toward the best alternative.

III. Stationary Choice Behavior

Changes in the value of the commodity stock between periods are driven by the flows of expenditure and consumption. We have already argued that choice, and hence expenditure, can be determined only up to a probability distribution. Using (2), the expected expenditure on commodity i during the period equals $y\Delta t\mathbb{P}_{i,t}$, while the corresponding consumption equals $p_i q_{i,t}\Delta t$. Therefore expected changes in the value of the commodity stock between intervals obey

$$(12)\qquad \Delta p_i Q_{i,t}=(y\mathbb{P}_{i,t}-p_i q_{i,t})\Delta t.$$

Since the individual aims to improve his utility level, $y\mathbb{P}_{i,t}$ in (12) can be interpreted as the value of the currently desired consumption rate on commodity i. Thus (12) implies that expected changes in the value of the commodity stock are driven by the difference between the values of currently desired and experienced consumption rates.

Through $\mathbb{P}_{i,t}$, (12) depends on the entire distribution of commodity stocks. Thus the dynamics of (12) may be complicated, and they may or may not lead the individual to stationary choice behavior, such that no further changes in the value of the commodity stock are observed between intervals.[8] We do not study these dynamics here. Instead, we confine our analysis to stationary choice behavior which corresponds to the time-invariant system

$$(13)\qquad y\mathbb{P}_i\Delta t=p_i q_i\Delta t.$$

Summing over one interval, (13) implies

$$(14)\qquad Y\mathbb{P}_i=Tp_i q_i.$$

Summing (14) over the n commodities, we obtain the stationary budget constraint

$$(15)\qquad y=\sum_{i=1}^{n}p_i q_i$$

since $\Sigma_i\mathbb{P}_i=1$. Furthermore, substituting (10) into (13), we have

$$(16)\qquad \Delta_i v-\Delta_1 v=\frac{1}{\mu}\ln\frac{p_i q_i}{p_1 q_1}\qquad i>1$$

[8]A stationary solution exists (see e.g., proposition 4 in Victor Ginsburgh et al. [1985]). Sufficient conditions for the stability of stationary states for systems more general than ours are given in proposition 5 of Ginsburgh et al. (1985).

IV. Relationship with Maximizing Behavior

Let us begin by defining a simple expression for true utility increments at the stationary state. We consider the decision of spending the unit of money on commodity i at the beginning of a period and its consequences for future periods along the stationary path. Denote the initial change in the stock of commodity i by $\Delta Q_i^{(0)}$, and denote the portion of $\Delta Q_i^{(0)}$ remaining after m periods by $\Delta Q_i^{(m)}$. Taking into account $\Delta Q_i^{(0)} = 1/p_i$ and (3), we have

$$\Delta Q_i^{(m)} = \frac{1}{p_i}(1 - c_i \Delta t)^m \qquad m = 0,1,2,\dots. \tag{17}$$

Using (4), the correspondence change in the consumption flow is

$$\Delta q_i^{(m)} = \frac{c_i}{p_i}(1 - c_i \Delta t)^m \qquad m = 0,1,2,\dots. \tag{18}$$

We require that the true utility increment takes fully into account the future consequences on utility of the initial spending decision:

$$\Delta_i v = \sum_{m=0}^{\infty} \Delta_i v^{(m)}. \tag{19}$$

We also assume that the function $v(q_1,\dots,q_n)$, which determines the true current utility generated by the n consumption rates, is differentiable, strictly increasing, and strictly quasi-concave. If we expand $v(\cdot)$ in Taylor series around q_i and retain only linear terms, we can express true utility increments as

$$\Delta_i v \cong \sum_{m=0}^{\infty} \Delta q_i^{(m)} v_i = \frac{c_i v_i}{p_i} \sum_{m=0}^{\infty} (1 - c_i \Delta t)^m = \frac{v_i}{p_i \Delta t} \tag{20}$$

where $v_i \equiv \partial v / \partial q_i$ and where the intermediate step follows from (18). From now on, we shall assume that (20) holds as an equality.

Introducing (20) in (16), we obtain

$$\frac{v_i}{p_i} - \frac{v_1}{p_1} = \frac{\Delta t}{\mu} \ln \frac{p_i q_i}{p_1 q_1} \qquad i > 1 \tag{21}$$

where v_i / p_i is the marginal utility per unit of expenditure. Equations (15) and (21) can be used to determine the stationary consumption rates $(\bar{q}_1,\dots,\bar{q}_n)$ where, from now on, an overbar will denote variables at stationary state. According to our interpretation of (12), stationary consumption rates also represent desired consumption rates: since, in this case, the aspirations of the individual match its experiences, there is no need for further change in the commodity stock. Solving (15) and (21), we have

$$\bar{q}_i = \bar{q}_i(y, \mathbf{p}; \mu) \tag{22}$$

where $\mathbf{p}$ is the vector of prices. Using (13) and (22), the corresponding stationary utility level can be written as

$$v(\bar{q}_1,\dots,\bar{q}_n) = v\left(\frac{y\bar{\mathbb{P}}_1}{p_1},\dots,\frac{y\bar{\mathbb{P}}_n}{p_n}\right) \equiv \bar{v}(y, \mathbf{p}; \mu). \tag{23}$$

When there is no ability to choose, (21) implies $p_i \bar{q}_i = p_j \bar{q}_j$ for all i and j, which is consistent with (11). That is, under complete inability to determine what constitutes a good choice, the individual allocates an equal amount of money per unit of time to all commodities. On the other hand, when the ability to choose is perfect, (21) implies that, for all i and j, at least one of the following conditions must hold: (a) $\bar{v}_i / p_i = \bar{v}_j / p_j$; (b) $\bar{q}_i = 0$; (c) $\bar{q}_j = 0$; (d) both $\bar{q}_i = 0$ and $\bar{q}_j = 0$. It follows immediately that the stationary consumption rates corresponding to a perfect ability to choose satisfy the necessary conditions of maximizing $v(\cdot)$ subject to $\Sigma_i p_i q_i \le y$, which is consistent with (11). Furthermore, it can be shown that the stationary consumption rates tend to the solution of the constrained maximization problem as $\mu \to \infty$, which implies the continuity of the stationary solutions for $\mu > 0$.

V. Satisficing Behavior

We now turn to a main result of our paper, namely, that the behavior of the individual under an imperfect ability to choose is *satisficing* in the sense that its stationary choice behavior corresponds to a level of utility that is below the maximum possible (Simon, 1955; Leibenstein, 1976 Ch. 5). In particular, we shall prove that $d\bar{v}/d\mu > 0$ for $0 < \mu < \infty$. Notice that, since utility is differentiable, strictly increasing, and strictly quasi-concave, any $(\bar{q}_1, \ldots, \bar{q}_n) \equiv \bar{\mathbf{q}}$ is equivalent to the solution of a standard utility maximization problem with adjusted income and prices. This happens because a single indifference surface passes through $\bar{\mathbf{q}}$ and, in turn, a single hyperplane is tangent to this surface at $\bar{\mathbf{q}}$. We call this an equivalent maximization problem (EMP). As $\bar{\mathbf{q}}$ changes smoothly, so do the income and prices of the EMP, given a particular good as the numeraire. Also notice that

$$(24)\quad \sigma^2 \equiv \sum_{i=1}^{n} \mathbb{P}_i \left(\frac{v_i}{p_i}\right)^2 - \left(\sum_{i=1}^{n} \mathbb{P}_i \frac{v_i}{p_i}\right)^2$$

can be interpreted as the variance of a discrete random variable having realizations v_i/p_i which occur with probability $\mathbb{P}_i$. Now since $\bar{v}(\cdot)$ can also be treated as the solution of an EMP, we may use the envelope theorem on the definition in (23) to obtain

$$(25)\quad \frac{d\bar{v}}{d\mu} = y\bar{\sigma}^2$$

which is positive provided that $\bar{\sigma}^2 \to 0$ only if $\mu \to \infty$. The qualification placed on this result is not restrictive: the only exception occurs when the conditions $v_i/p_i = v_j/p_j$ for all i and j are satisfied for any value of μ, in which case system (21) admits a solution independent of μ and defined by $p_i q_i = p_j q_j$ for all i and j. Then $\bar{\sigma}^2 = 0$ for all μ. In any other case, $\bar{\sigma}^2 \to 0$ only if $\mu \to \infty$.[9]

It is interesting to note the dependence of $d\bar{v}/d\mu$ in (25) on the distribution of marginal utilities. For a given ability to choose, larger variance of this distribution implies larger gains in stationary utility caused by an ability improvement. This agrees with intuition. Since an improvement in the ability to choose tightens the distribution of marginal allocation probabilities around better alternatives, the expected benefit of such an improvement will increase with a larger variance of the distribution. To fix ideas, imagine first a choice among commodities with very similar characteristics. Since, in this case, the choice is not particularly important, the benefit from an ability improvement is not large. Now imagine that the consumption of some commodities has health or safety implications: some could save your life, others could kill you. In this high-variance environment, one would expect that an ability improvement may be crucially important for the individual's well-being.

The analysis of demand functions and comparative statics for an exogenous ability to choose appears in Appendix A-1. Not surprisingly, this is a straightforward generalization of the standard analysis.

VI. The Case of Two Commodities

In this section we present two graphs of stationary choice behavior when there are only two commodities. The line AB in Figure 1 represents the stationary budget constraint. Point C, the point of tangency between the budget constraint and the highest possible difference curve I^∞, represents stationary consumption rates under a perfect ability to choose. Point D, where $p_1 q_1 =$

[9]Directly computing the sign of $d^2\bar{v}/d\mu^2$ shows $\bar{v}(\cdot)$ to be convex for small μ and concave for large μ. In consequence, the marginal return to improving the ability to choose first increases and then decreases. In particular, the concavity of the curve when μ is sufficiently large guarantees that $d\bar{v}/d\mu$ in (25) remains finite as $\mu \to \infty$.

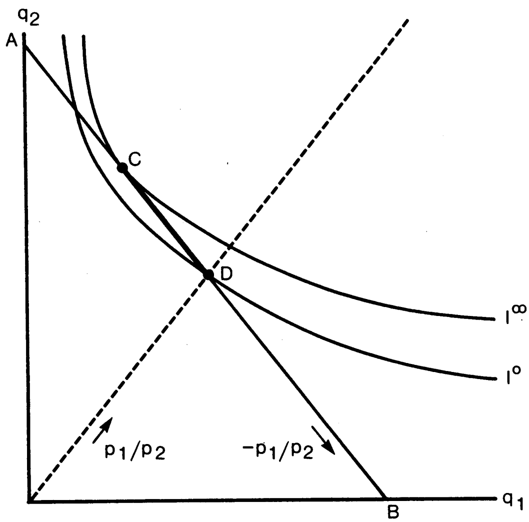

FIGURE 1. STATIONARY CONSUMPTION RATES

p_2q_2, represents stationary consumption rates under no ability to choose on a lower indifference curve I^0. In between these two indifference curves lies the range of stationary utility indifference curves as μ changes. Every particular indifference curve within that range intersects only once with the line segment CD to define an associated set of stationary consumption rates. Clearly, CD is the set of all possible stationary consumption rates. A dispersion of stationary consumption rates over CD implies different abilities to choose, where higher ability is located closer to C. Such dispersion can be observed in the third experiment of Herrnstein (1991). In that experiment, subjects had the same preferences, endowments, and access to information. Consequently, the dispersion of stationary states in that experiment implies different abilities to solve the same economic problem, that is, different abilities to choose.

An alternative, more detailed point of view stems from a direct study of (21) in the case of two commodities, namely,

$$\frac{v_2}{p_2} - \frac{v_1}{p_1} = \frac{\Delta t}{\mu} \ln \frac{y - p_1q_1}{p_1q_1}. \tag{26}$$

Figure 2A shows the right-hand side (RHS) of this equation for different values of μ. When μ approaches 0, the function tends to a vertical line crossing the axis q_1 at $y/2p_1$. On the other hand, when μ approaches infinity, the function tends to zero for every possible value of q_1 except for $q_1 = 0$ and $q_1 = y/p_1$. Intermediate values of μ will generate graphs between these two extremes.

We now proceed on the assumption that the left-hand side (LHS) of (20) satisfies[10]

$$\frac{d}{dq_1}\left(\frac{v_2}{p_2} - \frac{v_1}{p_1}\right) > 0. \tag{27}$$

Figure 2B displays four possible graphs of the LHS of (26) superimposed on the graphs of the associated RHS of Figure 2A. Points A and B represent corner solutions at $\lim \mu \to \infty$, while points C and D represent corresponding interior solutions. The solution D is independent of μ, a case which was identified as the only exception to $d\bar{v}/d\mu > 0$ in Section V. Notice that if $\mu < \infty$ then there can be no corner solution. Furthermore, with the exception of D, stationary consumption rates change monotonically as the ability to choose improves or deteriorates. It is evident from (21) that the absence of corner solutions for $\mu < \infty$ is generally true. Monotonicity, however, which must hold in both Figures 1 and 2A, does not apply in general.[11]

[10]When this assumption does not hold, the LHS of (26) cannot cross the horizontal axis of Figure 2B. Otherwise, the uniqueness of the stationary consumption rates for $\lim \mu \to \infty$ is violated. It follows that (27) rules out only some cases with corner solutions at $\lim \mu \to \infty$. Everything else, which represents the vast majority of cases, is consistent with (27).

[11]An example will suffice. Let $v(\mathbf{q}) \equiv q_1^\alpha q_2^\beta q_3^\gamma$, with q_1 the numeraire. Also let $y = 1$, $\alpha = 1/3$, and $\beta + \gamma = 2/3$. It follows that $\bar{q}_1 = 1/3$ for $\mu = 0$. Since we know that $\bar{q}_1$ attains the same value for $\mu = \infty$, it is enough to show that there exists $\mu^* > 0$ such that $\bar{q}_1 \neq 1/3$ for $\mu = \mu^*$. Applying (21), we conclude that $\bar{q}_1 = 1/3$ for $\mu = \mu^*$ is violated at $\mu^* = \bar{v}(\cdot)$ because it requires solving two independent equations for a single unknown. It is always possible to select such μ^* since $\bar{v}(\cdot)$ is continuous and bounded, and since $\mu > 0$.

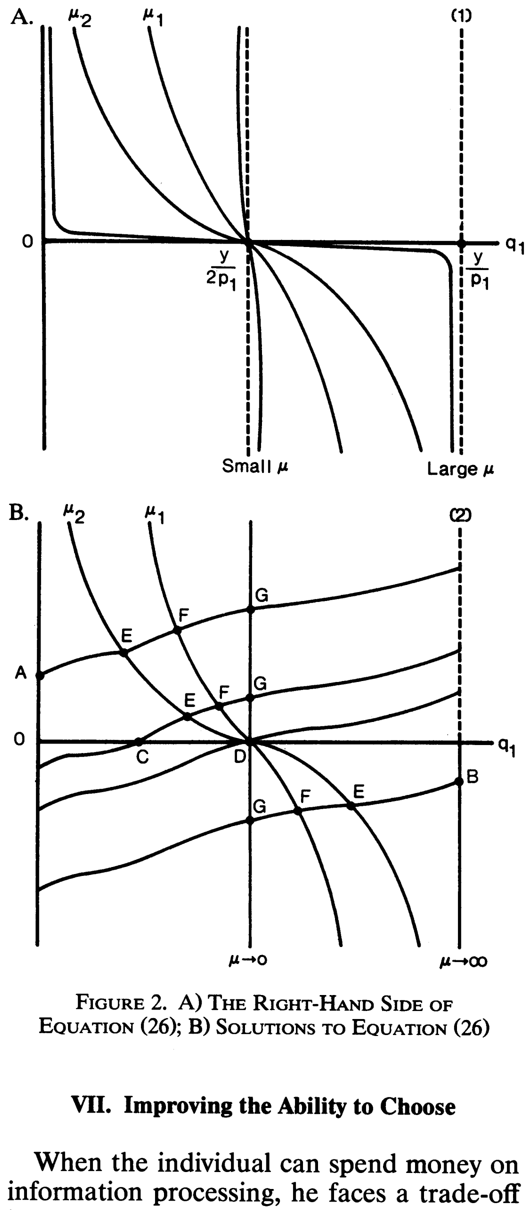

FIGURE 2. A) THE RIGHT-HAND SIDE OF EQUATION (26); B) SOLUTIONS TO EQUATION (26)

VII. Improving the Ability to Choose

When the individual can spend money on information processing, he faces a trade-off between his ability to choose and disposable income. Let $\mu(c)$ represent the ability to choose achieved at the expenditure rate c, where $\mu'(c) > 0$. Also let $\psi \equiv y + c$; that is, ψ is the total expenditure rate. The stationary level of utility $\bar{v}(\cdot)$ can now be written as $\bar{v}(\psi - c, \mathbf{p}; \mu(c))$ and, therefore,

$$(28) \qquad \frac{d\bar{v}}{dc} = \left.\frac{\partial \bar{v}}{\partial c}\right|_{\mu} + \left.\frac{\partial \bar{v}}{\partial \mu}\frac{d\mu}{dc}\right|_{\psi - c}.$$

The individual can aim toward the best level of ability as follows. At the beginning of each interval $[(k-1)T, kT]$, $k = 1, 2, \ldots$, the individual decides to spend at a rate c_k on information acquisition and processing, thus determining both the ability to choose $\mu_k \equiv \mu(c_k)$ and a corresponding stationary utility level $\bar{v}_k \equiv \bar{v}(\psi - c_k, \mathbf{p}; \mu(c_k))$ for the interval. We assume that all adjustments are fast, so that the individual enjoys $\bar{v}_k$ over the entire interval. For every interval, the decision of the individual about c_k is based on past performance: if his efforts to improve ability have reduced the stationary utility level, the individual will relapse to a lower level of effort; and if it pays to improve ability, the individual will intensify effort.

We begin with a stationary utility level $\bar{v}_0$, which corresponds to $c_0 = 0$. In the first interval the individual allocates $c_1 = h\bar{v}_0$ toward improving ability, where h is a constant. For $k \geq 2$, changes in c_k are determined by

$$(29) \qquad \Delta c_k = \begin{cases} h\Delta\bar{v}_{k-1} & \text{if sign } \Delta\bar{v}_{k-1} = \text{sign } \Delta c_{k-1} \\ -h\Delta\bar{v}_{k-1} & \text{otherwise.} \end{cases}$$

For h small enough, the simple adjustment process in (29) can lead the individual to an optimum. This is illustrated in Figure 3, which is a detailed version of Figure 1, where the discrete stationary path just described belongs to the continuous line DFE. This line represents the locus of stationary consumption rates as a function of the expenditure rate. A movement toward the left of the line corresponds to a higher expenditure rate and, therefore, to a lower budget constraint. It has been drawn so that, near zero ability to choose, improving ability also improves the stationary utility level. We also know that DFE passes through the origin (if you spend everything on improving your ability to choose, you can buy nothing else). Thus there must be at least one level of ability to choose that maximizes the stationary utility level. When DFE is strictly concave, as in Figure 3, the single optimum will be found at F, where DFE is tangent to the

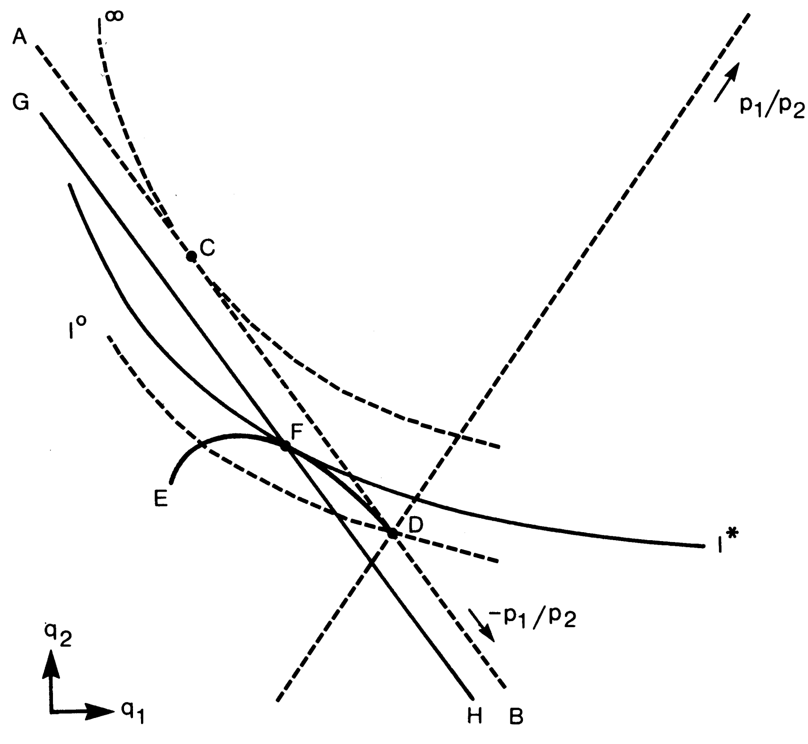

FIGURE 3. OPTIMAL ABILITY TO CHOOSE

indifference curve I*, and intersects with the budget constraint GH which is lower than the original AB. At the limit $h \to 0$, c^*, μ^*, and $\bar{v}^*$ will become independent of the interval, and a stationary state analogous to that of Section III will obtain. When DFE is not that well-behaved, the individual may attain only a local optimum.

Following our remarks in Section V, one should expect a higher marginal return from investment in the ability to choose for a larger variance in the distribution of marginal utilities. That is, individuals will benefit more by investing in ability improvements if decisions to be taken are more important. Once a stationary equilibrium has been achieved in which individuals have appropriately taken into account the trade-off between their ability to choose and disposable income, there is no way left to improve on stationary consumption rates. Further improvements can be made only if there are advantages to be explored in collective information gathering, for example, when the cost of information is very high for individuals as in personal-safety issues. Home-management courses, public warnings about cigarette use or about alcohol abuse, fire drills, and the like serve as examples of methods aiming at further improvements of the ability to choose. If the cost of such methods is equally borne by the individuals, $\mu(c)$ reflects returns to scale in collective information gathering. More generally, efforts to improve ability at the individual and the public levels seem complementary, and the optimal mix of these efforts largely remains an empirical question.

When individuals optimize their ability to choose, differences in ability will arise because of inherent differences in their information-processing capacity (intelligence, quality of memory, judgment, etc.). This happens because processing capacity represents an indivisible resource for individuals. Since some individuals are better instruments of choice than others, the utility-possibility frontier becomes asymmetric, and unless the Rawlsian principle applies, individuals should be treated unequally even if their preferences are identical.[12]

The analysis of demand functions for an endogenous ability to choose appears in Appendix A-2.

VIII. Paternalism

The adjustment process in (29) assumes no error in the decision to spend for improving the ability to choose. If, however, we admit the existence of errors in consumption choices, why preclude analogous errors in the context of (29)? For example, if spending c_k now increases μ only at the beginning of the next interval, someone following (29) will decide against investment to improve ability. A typical case is provided by a child who underestimates the future benefit of the current effort required for education. Since taxing the individual and then subsidizing ability improvements would improve welfare, we conclude that the first welfare theorem can fail under an imperfect ability to choose, even in the absence of externalities. With more than one individual and under the same myopic conditions, the second welfare theorem also fails because no redistribution can induce individuals to choose their optimal ability improvements. Such failures need not arise from a lack of information, but rather because individuals may not have the information-processing capacity necessary to equate the marginal cost of improving ability with the corresponding marginal benefit. For example, a child may have complete and accurate information provided by his parents, yet lack the judgment necessary to use the information appropriately to make the utility-maximizing decision.

There is a case for paternalism in our framework. Parents insisting that their child must go to school may well improve the child's welfare. Similarly, governments insisting that all children must be provided with medical care may well improve the welfare of children whose parents have a religious objection to medicine. Notice, however, that paternalism does not only require an "inferior" ability to choose from the subject and a "superior" ability to choose from the decision-maker, but also an ability for imposing coercive decisions. Thus, although paternalism in our framework cannot be rejected on the basis of consumer sovereignty as under a perfect ability to choose, it could still be rejected on the basis of an aversion to the coercive power it entails.

IX. Systematic Distortions in Perception

Advertisement in economics is modeled either as providing direct information about the product (Lester G. Telser, 1964; Gene M. Grossman and Carl Shapiro, 1984) or as creating an "image" for the product which can either serve as a signal of product quality (Paul Milgrom and John Roberts, 1986) or enter the utility function as a complement to the product advertised (Becker and Murphy, 1993).[13] Such models imply that advertisement can be good since it provides either useful information or utility, and they offer powerful ways of thinking about some advertisement-related phenomena. There are, however, those who believe that image advertisement can be bad since it is meant to manipulate consumers (John K. Galbraith, 1967). A standard example is provided by a number of competing firms which produce highly similar products and which use image advertisement to create artificial differentiation for their product. A

[12]This result is well known in urban economics (J. A. Mirrlees, 1972), where unequal treatment at the optimum arises because location quality is also indivisible. For a diagrammatic exposition, in which differences in location advantage can be reinterpreted as differences in processing capacity, see David Levhari et al. (1978).

[13]An exception is provided by Avinash Dixit and Victor Norman (1978), who modeled advertisement as changing preferences.

$$(32)\qquad \frac{v_2}{p_2}-\frac{v_1}{p_1}=\frac{\Delta t}{\mu}\ln\left(\left[\frac{y}{2}-p_1q_1+\sqrt{\left(\frac{y}{2}-p_1q_1\right)^2+e^{-(b_1+b_2)}p_1q_1(y-p_1q_1)}\right]\Big/p_1q_1e^{-b_1}\right)$$

firm in this context aims to bias consumers in favor of its product even though there is no intrinsic reason for preference. More generally, bias from manipulative advertisement can arise only in the context of errors made because of an imperfect ability to choose.[14] As the ability to choose improves, manipulative advertisement should become less effective, and hence bias should become smaller. At the limit, where the ability to choose is perfect, the individual should not rely any longer on biased information, but only on the intrinsic character of products. We express this possibility by assuming that manipulative advertisement in favor of commodity i can lead to errors having a positive expectation $E(\varepsilon_{i,t})=b_i$, where b_i is the bias associated with that commodity. Our previous remarks suggest that (9) can be generalized as

$$(30)\qquad \varepsilon_{i,t}=\frac{1}{\mu}(\varepsilon+b_i)$$

which implies that the marginal allocation probabilities are now given by

$$(31)\qquad \mathbb{P}_{i,t}=\frac{\exp(\mu\Delta_{i,t}v+b_i)}{\sum_{j=1}^{n}\exp(\mu\Delta_{j,t}v+b_j)}.$$

We consider only the case of two commodities. Introducing (31) in (13), and taking into account (20), we have equation (32) above at steady-state, which reduces to (21) for $b_1=b_2=0$. Figure 4A shows the RHS of this equation for different values of μ. Relative to Figure 2A, it is seen that systematic distortions in perception shift the point common to all curves along the abscissa. If $b_2<(>)\ b_1$, the common point shifts to the right (left) of $y/2p_1$. In the first case, shown in Figure 2A, the bias encouraging adjustments toward the first commodity is stronger than that encouraging adjustments away from it. Furthermore, the length of the shift depends on the difference between b_1 and b_2. Thus the amount of effort spent on advertisement is not important—only the differential effort: when $b_1=b_2$, biases cancel each other and Figure 4A reduces to Figure 2A. As b_1-b_2 increases, the RHS of (32) changes as in Figure 4B. Two possible graphs of the LHS are also shown in that figure. Applying the logic of Figure 2B, the stationary consumption rate of a better-advertised commodity increases. In combination, parts A and B of Figure 4 suggest how, as the ability to choose improves, systematic distortions become less important until, under perfect ability to choose, they cannot any longer affect marginal allocation adjustments for any degree of differential effort.

There is a scope for control of manipulative advertisement in our framework. In a world of perfect ability to choose, as that of the standard model, there is no rationale for the regulation of the effects of advertisement on the individual since advertisement is either good or it is discarded. However, in a world of imperfect ability to choose, laws against false advertisement and regulation of some advertisement may be accounted for on the basis of removing bias that decreases utility. This seems particularly true for commodities such as tobacco and alcohol, where errors may become dangerous, as well as for advertisement aimed specifically at low-ability groups such as young children. There is also a scope for the public manipulation of bias. For example, in the

[14] In models of rational expectations, errors are unbiased because, even though individuals lack information, they are otherwise perfect instruments of choice; hence they do not make systematic errors.

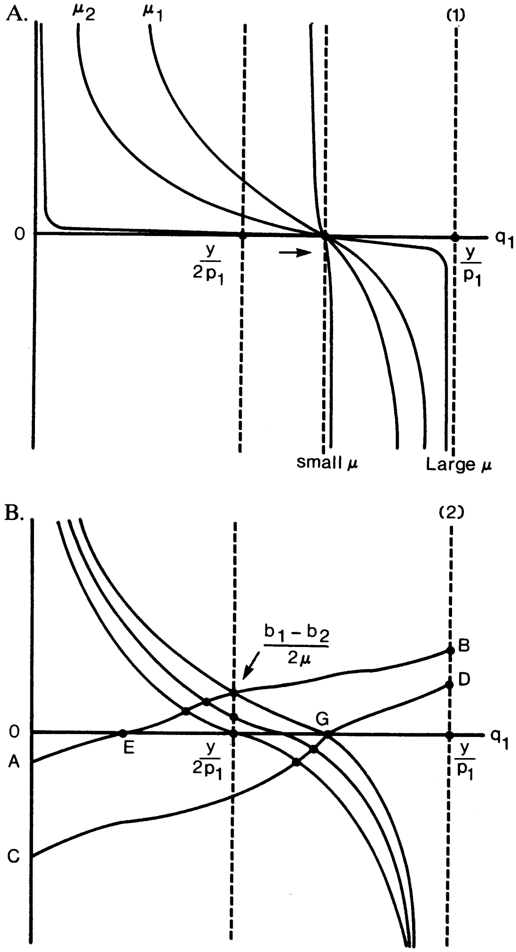

FIGURE 4. A) THE RIGHT-HAND SIDE OF EQUATION (32); B) SOLUTIONS TO EQUATION (32)

presence of externalities, correction could involve public advertisement instead of Pigouvian taxes and subsidies. Furthermore, given a particular ability to choose, Figure 4B indicates that bias may either decrease (line AB) or increase (line CD) the stationary level of utility. In particular, there is $b_1 - b_2$ which will generate the optimal allocation at stationary state. It follows that the quantities b_i can be used as controls. Unlike firms, which waste resources by trying to overcome the advertising effects of their competitors, government in our model can simply determine $b_1 > 0$ and $b_2 < 0$ for which the stationary consumption rates in the case of line CD are found at G. In any case, if no other advertisement is allowed, the planner can determine a combination of biases such that $b_1^* - b_2^*$ will accomplish G at minimum cost. Only when the ability to choose is perfect will it be unnecessary to influence the individual. Since both the ability to choose and the pattern of systematic distortions to perception can be affected, the problem is to find a best combination of ability-improvement programs and systematic distortion strategies. One cannot depend exclusively upon ability-improvement programs, since it becomes more costly to raise already higher ability while, at the same time, the corresponding savings on systematic manipulation strategies decline. Finally, public manipulation of bias can become bad, as suggested by propaganda.

X. Optimal Product Differentiation

We have already provided some examples indicating how standard welfare results can change under an imperfect ability to choose. In this section we reinforce this implication by developing a model in which we specify precisely how a welfare result changes with the degree of ability. We examine optimal product differentiation using the simplest model consistent with our framework. There is a population of individuals identical with respect to income and ability to choose. Following the tradition of Harold Hotelling (1929), we assume that this population is continuously distributed at a unit density over a line interval [0,1] representing the space of the single characteristic over which products are defined. Location on that space determines the preferences of those located there because the ideal product for an individual located at $x \in [0,1]$ has a characteristic of size x. There are only two products which are produced at the same, fixed marginal cost and sold at the same price. We consider a single interval $[0,T]$, at the beginning of which every individual spends a disposable income of one unit on a single product, which it consumes at a uniform rate $1/T$. In terms of Section I, $T = \Delta t = c^{-1}$. For an individual located at x, the utility of spending the unit of money on product i is simply

$$(33) \qquad v_i(x) = -\theta|x - x_i|$$

where $\theta > 0$ measures the intensity of preference for the ideal product and $x_i \in [0,1]$ is the address of product i. Using (10), the probability that an individual located at x will choose product i is

$$(34) \quad \mathbb{P}_i(x) = \frac{\exp(-\mu\theta|x - x_i|)}{\sum_{j=1}^{2} \exp(-\mu\theta|x - x_j|)}.$$

The expected utility of that individual can be written as

$$(35) \quad V(x, x_1, x_2) \equiv -[\theta|x - x_1|\mathbb{P}_1(x) + \theta|x - x_2|\mathbb{P}_2(x)]$$

and the total expected utility in the system is given by

$$(36) \quad W(x_1, x_2) \equiv \int_0^1 V(x, x_1, x_2)\,dx.$$

Since firms produce under the same linear technology and sell at the same price, and since total demand is fixed, we may disregard profits in constructing a socially optimal rule. Therefore our problem is to determine the optimal products (x_1^*, x_2^*), so that $W(\cdot)$ is maximized for a given ability to choose. In Appendix B-1 we prove that the optimal products are symmetrically located on [0,1] and behave as in Figure 5. When the ability to choose is perfect, we obtain the standard solution on the first and third quarters of the characteristic space (see e.g., Melvin L. Greenhut et al., 1987). As the ability to choose declines, optimal products become increasingly similar, and Hotelling's principle of minimum differentiation is eventually restored when the ability to choose becomes sufficiently low (also see de Palma et al., 1985). This could be interpreted as a justification of the familiar belief that less product variety can be better.

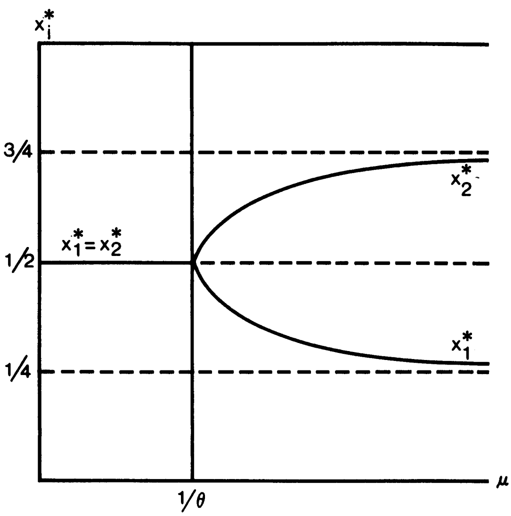

FIGURE 5. OPTIMAL PRODUCTS

For stronger intensity of preference, the range over which the principle is restored becomes smaller. Thus intensity of preference, as ability to choose, requires more differentiation. These conclusions are similar to the optimal location policies developed by Simon P. Anderson and de Palma (1988). We compare the two approaches in Appendix B-2.

XI. Concluding Remarks

The power and the success of perfectly rational behavior as a model of choice is to be found in the observation that many real events can be explained by assuming that individuals behave *as if* they have a perfect ability to choose. Yet there are phenomena for which explanations alternative to those provided within the context of perfectly rational behavior might appear closer to experience in some respects; and there are cases in which the "as if" principle itself breaks down. A positive example of "as if" failure is provided by the Herrnstein (1991) experiments in Section I, and a normative example is provided by the optimal product differentiation of Section X. Our basic premise, which provided the foundation for our model, is that those difficulties arise because the ability to choose often affects behavior in ways that cannot be dismissed as insignificant. With this in mind, we have strived to attain some meaningful balance between descriptive accuracy and analytical tractability. On the one hand, our model comes closer to observation than the model of perfectly rational behavior. On the other, modeling such behavior seems fruitful: the limited experience presented here indicates that our model can offer some analytical results which provide new insights.

Appendix A: Demand Functions

1. *Exogenous Ability To Choose*

At stationary consumption,

$$\text{(A1)} \qquad y\bar{\mathbb{P}}_i = p_i\bar{q}_i,$$

Total differentiation yields

$$\text{(A2)} \qquad (\mathbf{A}-\mathbf{I}\cdot\mathbf{p})\,d\bar{\mathbf{q}} = -\,y\frac{\partial\bar{\mathbb{P}}}{\partial\mu}\,d\mu-(\mathbf{B}-\mathbf{I}\cdot\bar{\mathbf{q}})\,d\mathbf{p}-\bar{\mathbb{P}}\,dy$$

where $\mathbf{A}$ and $\mathbf{B}$ are $n\times n$ matrices with elements

$$\text{(A3)} \qquad a_{ij}\equiv y\sum_{k=1}^{n}\frac{\partial\bar{\mathbb{P}}_i}{\partial\bar{v}_k}\frac{\partial\bar{v}_k}{\partial\bar{q}_j}$$

$$b_{ij}\equiv y\frac{\partial\bar{\mathbb{P}}_i}{\partial\bar{q}_j}$$

respectively, $\bar{\mathbb{P}}$ is the vector of stationary probabilities, and $\partial\bar{\mathbb{P}}/\partial\mu$ is the vector with components $\partial\bar{\mathbb{P}}_i/\partial\mu$. Equation (A2) describes the local changes in the stationary demand caused by any combination of local changes in the ability to choose, prices, and the expenditure rate. The effect on the equilibrium allocation of a change in the ability to choose when prices and the expenditure rate remain fixed is given by

$$\text{(A4)} \qquad d\bar{\mathbf{q}}=-(\mathbf{A}-\mathbf{I}\cdot\mathbf{p})^{-1}y\frac{\partial\bar{\mathbb{P}}}{\partial\mu}\,d\mu$$

provided that the matrix $\mathbf{A}-\mathbf{I}\cdot\mathbf{p}$ is nonsingular.

Consider now the associated EMP. For any $(y,\mathbf{p};\mu)$ there is $(\bar{y},\bar{\mathbf{p}})$ such that $\bar{\mathbf{q}}(y,\mathbf{p};\mu)$ is the solution to the problem of maximizing $v(\cdot)$ subject to $\sum_i\bar{p}_iq_i\leq\bar{y}$. Thus at $\bar{\mathbf{q}}+d\bar{\mathbf{q}}$ there is another $(\bar{y}+d\bar{y},\bar{\mathbf{p}}+d\bar{\mathbf{p}})$ such that

$$\text{(A5)} \qquad y\frac{\partial\bar{\mathbb{P}}}{\partial\mu}\,d\mu=(\mathbf{B}-\mathbf{I}\cdot\bar{\mathbf{q}})/d\bar{\mathbf{p}}-\bar{\mathbb{P}}\,d\bar{y}.$$

Equations (A4) and (A5) imply

$$\text{(A6)} \qquad d\bar{\mathbf{q}}=-(\mathbf{A}-\mathbf{I}\cdot\bar{\mathbf{p}})^{-1}\Big[(\mathbf{B}+\bar{\mathbb{P}}\cdot\bar{\mathbf{q}}'-\mathbf{I}\cdot\bar{\mathbf{q}})\,d\bar{\mathbf{p}}+\bar{\mathbb{P}}(d\bar{y}-\bar{\mathbf{q}}'\cdot d\bar{\mathbf{p}})\Big]$$
$$\equiv\frac{\partial\bar{\mathbf{q}}}{\partial\bar{\mathbf{p}}}\bigg|_{\bar{v}}\cdot d\bar{\mathbf{p}}+\frac{\partial\bar{\mathbf{q}}}{\partial\bar{y}}(d\bar{y}-\bar{\mathbf{q}}'\cdot d\bar{\mathbf{p}})$$

where $\partial\bar{\mathbf{q}}/\partial\bar{\mathbf{p}}$ is the $n\times n$ matrix with elements $\partial\bar{q}_i/\partial\bar{p}_j$, $\partial\bar{\mathbf{q}}/\partial\bar{y}$ is the vector with components $\partial\bar{q}_i/\partial\bar{y}$, and $\bar{\mathbf{q}}'$ is the transpose of $\bar{\mathbf{q}}$. The RHS of (A6) represents the decomposition of a change in the ability to choose into substitution and income effects of the EMP. How $\bar{y}$ and $\bar{\mathbf{p}}$ vary with μ can be determined through (A5), keeping a particular commodity as the numeraire.

Figure A1 illustrates those points in the case of two commodities. AB corresponds to the stationary budget constraint determined by the fixed y and $\mathbf{p}$. As in Figure 1, CD is the set of all possible stationary consumptions, where C corresponds to perfect ability and D to zero ability. Let E represent $\bar{\mathbf{q}}(y,\mathbf{p};\mu+d\mu)$ and let F represent $\bar{\mathbf{q}}(y,\mathbf{p};\mu)$. Both can be thought of as solutions to an EMP, the former with budget constraint GH and the latter with budget constraint JK. Both these new constraints involve changes in prices and the expenditure rate described by $d\bar{\mathbf{p}}$ and $d\bar{y}$ in (A5). As the ability to choose deteriorates, the stationary consumption moves from point E to point F on a lower indifference curve.

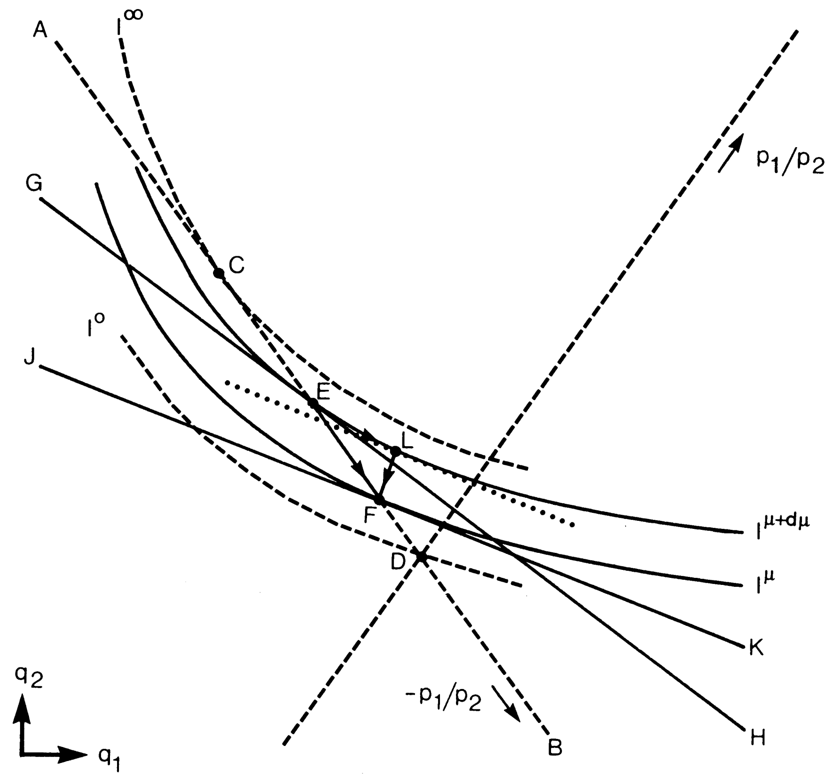

FIGURE A1. INCOME AND SUBSTITUTION EFFECTS

This movement can be decomposed into two components. The first part, EL, corresponds to the substitution effect of (A6) with L determined as the point of tangency between the higher indifference curve and the parallel to the budget constraint JK. The second, LF, corresponds to the income effect of (A6).

2. *Endogenous Ability To Choose*

The impact of improving ability on stationary consumption can be determined through a modification of (A2) Holding only prices fixed,

$$\text{(A7)}\quad \frac{d\bar{\mathbf{q}}}{dc}=(\mathbf{A}-\mathbf{I}\cdot\mathbf{p})^{-1}\cdot\bar{\mathbb{P}} -(\mathbf{A}-\mathbf{I}\cdot\mathbf{p})^{-1}(y-c)\frac{\partial\bar{\mathbb{P}}}{\partial\mu}\frac{d\mu}{dc}$$

where $d\bar{\mathbf{q}}/dc$ is the vector with components $d\bar{q}_i/dc$, and where the subscripts denoting an interval m have been omitted. Similarly to the case of utility in (28), the first term on the RHS of (A7) represents the change in stationary consumption due to a smaller disposable income (with the ability to choose held constant), while the second term represents the corresponding change due to an improved ability (with the disposable income held constant).

Figure A2 illustrates (28) and (A7) in the case of two commodities. Its structure and notation resemble those of Figure A1. Let AB represent the stationary budget constraint when $c=0$. Suppose now that the initial position is at E, where the individual spends a positive amount c to maintain a particular ability to choose and, consequently, has a stationary budget constraint GH lower than AB. In contrast, the stationary utility level lies on an indifference curve

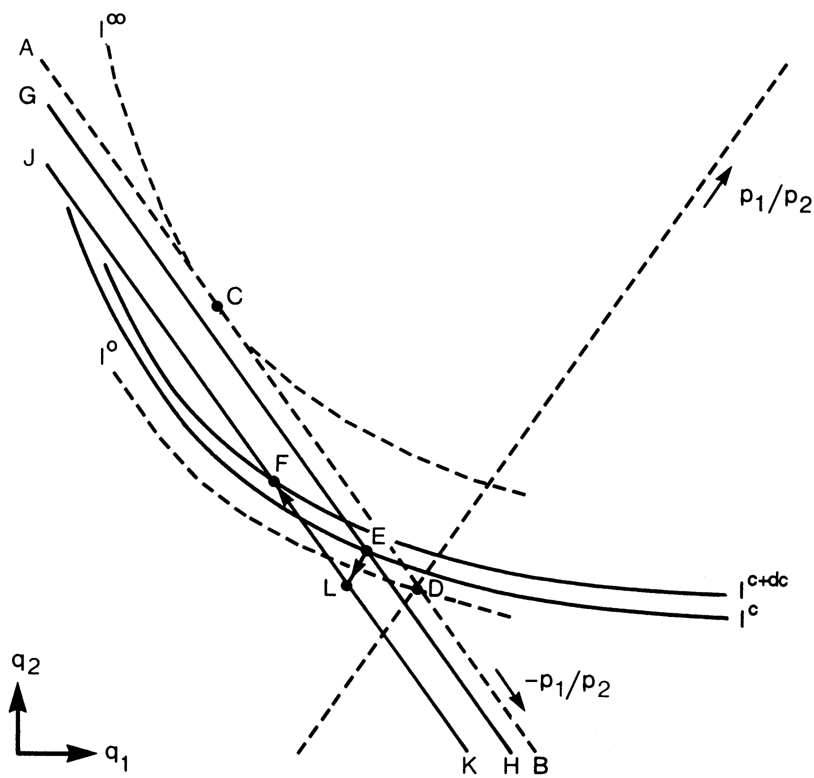

FIGURE A2. INCOME AND ABILITY EFFECTS

I^c which is higher than I^0 because, here, the improvement in ability dominates the corresponding loss of disposable income. The same continues to be true for further ability improvements in our example. Increasing c by dc results in the stationary consumption F, which lies at the intersection between a lower budget constraint JK and a higher difference curve I^{c+dc}. The movement from E to F can be partitioned into two components. The first, EL, corresponds to the first term on the RHS of (A7) and the second, LF, corresponds to the second term.

APPENDIX B: DETAILS ON OPTIMAL PRODUCT DIFFERENTIATION

Some calculations, which have not been included in this appendix, are available from the authors upon request.

1. *Optimal Products*

We first prove that the optimal products are symmetric in [0, 1]. Let

$$w_i \equiv \int_0^1 |x - x_i| \mathbb{P}_i \, dx. \tag{B1}$$

Since $W(\cdot) = -\theta \Sigma_i w_i$, maximizing $W(\cdot)$ is equivalent to minimizing $\Sigma_i w_i$ with first-order conditions given by $\partial(\Sigma_i w_i)/\partial x_j = 0$ for $j = 1, 2$. Now using

$$w_1 = \int_0^{x_1} (x_1 - x)\mathbb{P}_1^1 \, dx + \int_{x_1}^{x_2} (x - x_1)\mathbb{P}_1^2 \, dx + \int_{x_2}^{1} (x - x_1)\mathbb{P}_1^3 \, dx \tag{B2}$$

where $\mathbb{P}_1^1$, $\mathbb{P}_1^2$, and $\mathbb{P}_1^3$ represent $\mathbb{P}_1$ over the subintervals $[0, x_1)$, $[x_1, x_2)$ and $[x_2, 1]$, respectively. Differentiating (B2), we obtain

$$\text{(B3)}\quad \frac{\partial w_1}{\partial x_1} = \int_0^{x_1} \mathbb{P}_1^1\,dx - \int_{x_1}^{x_2} \mathbb{P}_1^2\,dx - \int_{x_2}^{1} \mathbb{P}_1^3\,dx + \int_0^{x_1} (x_1 - x)\frac{\partial}{\partial x_1}\mathbb{P}_1^1\,dx + \int_{x_1}^{x_2} (x - x_1)\frac{\partial}{\partial x_1}\mathbb{P}_1^2\,dx + \int_{x_2}^{1} (x - x_1)\frac{\partial}{\partial x_1}\mathbb{P}_1^3\,dx.$$

Taking into account that

$$\text{(B4)}\quad \frac{\partial}{\partial x_1}\mathbb{P}_1^1 = -\mu\theta\mathbb{P}_1^1(1-\mathbb{P}_1^1)$$

$$\frac{\partial}{\partial x_1}\mathbb{P}_1^k = \mu\theta\mathbb{P}_1^k(1-\mathbb{P}_1^k) \qquad k = 2,3$$

equation (B3) reduces to

$$\text{(B5)}\quad \frac{\partial w_1}{\partial x_1} = \int_0^{x_1} \mathbb{P}_1\,dx - \int_{x_2}^{1} \mathbb{P}_1\,dx + \mu\theta\int_0^{1} (x - x_1)\mathbb{P}_1\mathbb{P}_2\,dx.$$

Similarly,

$$\text{(B6)}\quad \frac{\partial w_2}{\partial x_1} = -\mu\theta + \mu\theta\int_{x_1}^{x_2} (x - x_2)\mathbb{P}_1\mathbb{P}_2\,dx - \mu\theta\int_{x_2}^{1} (x - x_2)\mathbb{P}_1\mathbb{P}_2\,dx.$$

Adding (B5) and (B6), and taking into account that $\mathbb{P}_i$ is symmetric with respect to $(x_1 + x_2)/2$, we obtain

$$\text{(B7)}\quad \frac{\partial}{\partial x_1}\sum_{i=1}^{2} w_i = \int_0^{x_1} \mathbb{P}_1\,dx - \int_{x_1}^{1} \mathbb{P}_1\,dx + \mu\theta(x_2 - x_1)\left(\int_0^{x_1} \mathbb{P}_1\mathbb{P}_2\,dx + \int_{x_2}^{1} \mathbb{P}_1\mathbb{P}_2\,dx\right) = 0.$$

The same procedure, applied to the address of the second product, leads to

$$\text{(B8)}\quad \frac{\partial}{\partial x_2}\sum_{i=1}^{2} w_i = \int_0^{x_2} \mathbb{P}_2\,dx - \int_{x_2}^{1} \mathbb{P}_2\,dx - \mu\theta(x_2 - x_1)\left(\int_0^{x_1} \mathbb{P}_1\mathbb{P}_2\,dx + \int_{x_2}^{1} \mathbb{P}_1\mathbb{P}_2\,dx\right) = 0.$$

Finally, adding (B7) and (B8), we conclude that

$$\text{(B9)}\qquad x_2^* = 1 - x_1^*$$

as required for symmetry.

We next examine the behavior of optimal products in relation to θ and μ. Upon substitution of (B9) into (B7) and taking into account the symmetry of $\mathbb{P}_i$ when the two product addresses are symmetric, the first-order condition (FOC) of the optimization problem can be written as

$$\text{(B10)}\quad \text{FOC} \equiv 4\mu\theta\mathbb{P}_1^1(1-\mathbb{P}_1^1)\left(\frac{1}{2} - x_1\right)x_1 + \left(2\mathbb{P}_1^2 x_1 - \frac{1}{2}\right) = 0$$

where

(B11) $\mathbb{P}_1^1 = \{1+\exp[\mu\theta(2x_1-1)]\}^{-1}$.

The corresponding second-order condition (SOC) is

$$\text{(B12)}\quad \text{SOC} \equiv 2\mathbb{P}_1^1\Big\{1-2\mu\theta(1-\mathbb{P}_1^1) \times\left[3x_1-\frac{1}{2}+2\mu\theta\left(\frac{1}{2}-x_1\right)(1-2\mathbb{P}_1^1)\right]\Big\} \geq 0.$$

At $x_1 = 1/2$ we have $\mathbb{P}_1^1 = 1/2$. Hence, for the same location, FOC = 0, and SOC $= 1 - \mu\theta$. It follows that $x_1 = 1/2$ corresponds to a local maximum of $W(\cdot)$ for $\mu \leq 1/\theta$ and to a local minimum for $\mu > 1/\theta$. We prove below that the maximum is global. Therefore $x_1^* = 1/2$ if and only if $\mu \leq 1/\theta$. For $\mu > 1/\theta$, x_1^* also corresponds to a global maximum of $W(\cdot)$. Total differentiation of (B10) further yields that x_1^* is strictly decreasing and convex in μ for $\mu > 1/\theta$. Hence the minimum occurs at $\mu = \infty$. In order to evaluate this minimum notice that $\lim \mathbb{P}_1^1 = 1$ and $\lim \mu\mathbb{P}_1^1 = 0$ for $\mu \to \infty$. These, in conjunction with the FOC, imply $\lim_{\mu\to\infty} x_1^* = 1/4$.

It remains to prove that the result is global. Using $x_1 \equiv 1/2 - z$, after some calculations, we can write (B10) as

$$\text{(B13)}\quad z[(\mu\theta-1)-2\mu\theta z] = \left(z-\frac{1}{4}\right)\exp(2\mu\theta z)+\frac{1}{4}\exp(-2\mu\theta z).$$

When $\mu \leq 1/\theta$, the LHS of (B13) is null for $z = 0$ and negative for $z > 0$, while the RHS of (B13) is null for $z = 0$ and positive for $z > 0$. In consequence (B13) admits a unique solution $z = 0$. This implies that $x_1^* = x_2^* = 1/2$ corresponds to the global maximum of $W(\cdot)$. We now examine the case $\mu > 1/\theta$. The LHS of (B13) is a concave parabola as shown in Figure B1, where LHS(1/2) < 0. After some calculations, we also establish that the RHS of (B13) is a convex function as shown in Figure B1 where RHS(1/2) > 0. Since, in Figure B1, $z = 0$ is a solution to

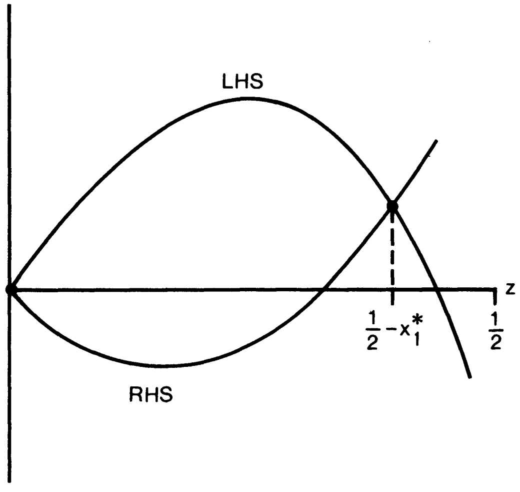

FIGURE B1. SOLUTIONS TO EQUATION (B13)

(B13) that corresponds to a local minimum of $W(\cdot)$, the only other solution must correspond to a global maximum.

2. *Anderson and de Palma (1988)*

In order to determine optimal location policies analogous to our optimal design policies, Anderson and de Palma (1988) employed the consumer-surplus function

$$\text{(B14)}\quad \text{CS}(x_1, x_2) \equiv \frac{1}{\mu}\int_0^1 \ln\left(\sum_{i=1}^{2}\exp[\mu\theta|x-x_i|]\right)$$

which was derived for the logit model by Kenneth A. Small and Harvey S. Rosen (1981). Taking into account symmetry, the first- and second-order conditions are

$$\text{(B15)}\quad \text{FOC} \equiv 2\mu\theta\left(\frac{1}{2}-x_1\right)+\ln(4x_1-1) = 0$$

$$\text{(B16)}\quad \text{SOC} \equiv 4-2\mu\theta(4x_1-1) \geq 0.$$

At $x_1 = 1/2$ we have FOC = 0 and SOC $= 2 - \mu\theta$. It follows that $x_1 = 1/2$ corresponds to a local maximum of $\text{CS}(\cdot)$ for $\mu \leq 2/\theta$, and to a local minimum for $\mu > 2/\theta$. As in our case, the maximum is also global. Thus the range over which

Hotelling's principle of minimum differentiation holds in Anderson and de Palma (1988) is twice our range. The remaining conclusions are exactly analogous in both cases.

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